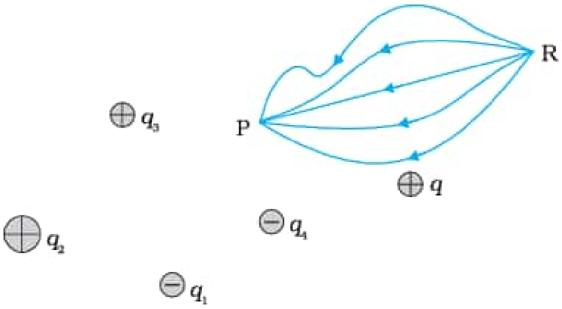

FIGURE 2.2 Work done on a test charge q by the electrostatic field due to any given charge configuration is independent of the path, and depends only on its initial and final positions.

In other words, the electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point.

The qualifying remarks made earlier regarding potential energy also apply to the definition of potential. To obtain the work done per unit test charge, we should take an infinitesimal test charge δq, obtain the work done δW in bringing it from infinity to the point and determine the ratio $\delta W/\delta q$. Also, the external force at every point of the path is to be equal and opposite to the electrostatic force on the test charge at that point.

2.3 Potential due to a Point Charge

Consider a point charge Q at the origin (Fig. 2.3). For definiteness, take Q to be positive. We wish to determine the potential at any point P with position vector $\mathbf{r}$ from the origin. For that we must calculate the work done in bringing a unit positive test charge from infinity to the point P. For $Q > 0$, the work done against the repulsive force on the test charge is positive. Since work done is independent of the path, we choose a convenient path – along the radial direction from infinity to the point P.

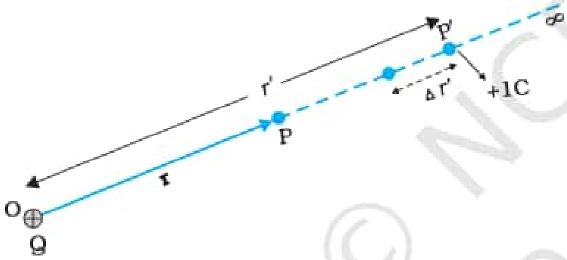

FIGURE 2.3 Work done in bringing a unit positive test charge from infinity to the point P, against the repulsive force of charge Q ($Q > 0$), is the potential at P due to the charge Q.

At some intermediate point P′ on the path, the electrostatic force on a unit positive charge is

$$\frac{Q \times 1}{4\pi\varepsilon_0 r'^2}\,\hat{\mathbf{r}}' \qquad (2.5)$$

where $\hat{\mathbf{r}}'$ is the unit vector along OP′. Work done against this force from $\mathbf{r}'$ to $\mathbf{r}' + \Delta\mathbf{r}'$ is

$$\Delta W = -\frac{Q}{4\pi\varepsilon_0 r'^2}\,\Delta r' \qquad (2.6)$$

The negative sign appears because for $\Delta r' < 0$, ΔW is positive. Total work done (W) by the external force is obtained by integrating Eq. (2.6) from $r' = \infty$ to $r' = r$,

$$W = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2}\,dr' = \left.\frac{Q}{4\pi\varepsilon_0 r'}\right|_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0 r} \qquad (2.7)$$

This, by definition is the potential at P due to the charge Q

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \qquad (2.8)$$

Equation (2.8) is true for any sign of the charge Q, though we considered $Q > 0$ in its derivation. For $Q < 0$, $V < 0$, i.e., work done (by the external force) per unit positive test charge in bringing it from infinity to the point is negative. This is equivalent to saying that work done by the electrostatic force in bringing the unit positive charge form infinity to the point P is positive. [This is as it should be, since for $Q < 0$, the force on a unit positive test charge is attractive, so that the electrostatic force and the displacement (from infinity to P) are in the same direction.] Finally, we note that Eq. (2.8) is consistent with the choice that potential at infinity be zero.

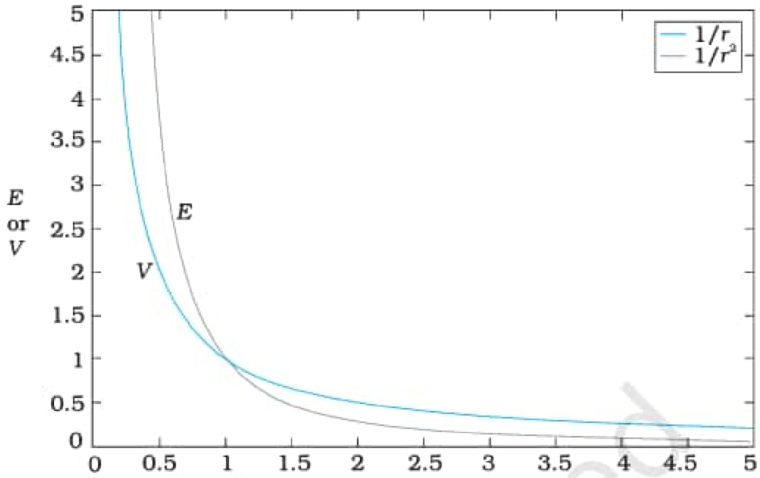

FIGURE 2.4 Variation of potential V with r [in units of $(Q/4\pi\varepsilon_0)$ m^{-1}] (blue curve) and field with r [in units of $(Q/4\pi\varepsilon_0)$ m^{-2}] (black curve) for a point charge Q.

Figure (2.4) shows how the electrostatic potential ($\propto 1/r$) and the electrostatic field ($\propto 1/r^2$) varies with r.

Example 2.1

(a) Calculate the potential at a point P due to a charge of 4×10^{-7}C located 9 cm away.

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Solution

(a) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = 9\times10^{9}\,\text{Nm}^2\,\text{C}^{-2} \times \dfrac{4\times10^{-7}\,\text{C}}{0.09\,\text{m}}$

$= 4 \times 10^{4}$ V

(b) $W = qV = 2 \times 10^{-9}\text{C} \times 4 \times 10^{4}\text{V}$

$= 8 \times 10^{-5}$ J

No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along **r** and another perpendicular to **r**. The work done corresponding to the later will be zero.

EXAMPLE 2.1

2.4 Potential due to an Electric Dipole

As we learnt in the last chapter, an electric dipole consists of two charges q and $-q$ separated by a (small) distance $2a$. Its total charge is zero. It is characterised by a dipole moment vector **p** whose magnitude is $q \times 2a$ and which points in the direction from $-q$ to q (Fig. 2.5). We also saw that the electric field of a dipole at a point with position vector **r** depends not just on the magnitude r, but also on the angle between **r** and **p**. Further,

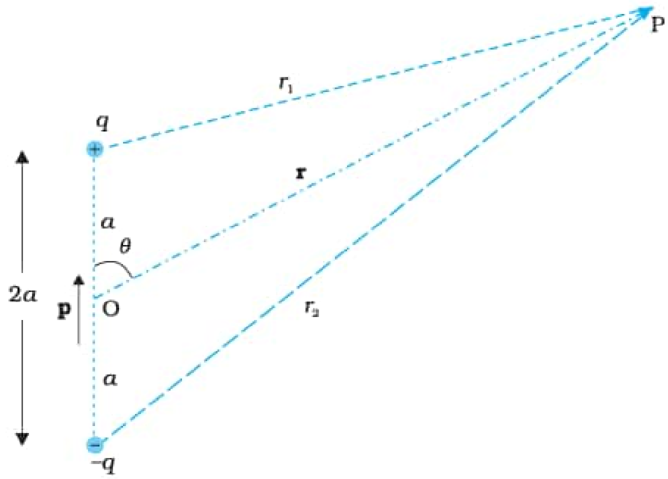

FIGURE 2.5 Quantities involved in the calculation of potential due to a dipole.

the field falls off, at large distance, not as $1/r^2$ (typical of field due to a single charge) but as $1/r^3$. We, now, determine the electric potential due to a dipole and contrast it with the potential due to a single charge.

As before, we take the origin at the centre of the dipole. Now we know that the electric field obeys the superposition principle. Since potential is related to the work done by the field, electrostatic potential also follows the superposition principle. Thus, the potential due to the dipole is the sum of potentials due to the charges q and $-q$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) \tag{2.9}$$

where r_1 and r_2 are the distances of the point P from q and $-q$, respectively.

Now, by geometry,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_2^2 = r^2 + a^2 + 2ar\cos\theta \tag{2.10}$$

We take r much greater than a ($r >> a$) and retain terms only upto the first order in a/r

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)$$

$$\cong r^2\left(1 - \frac{2a\cos\theta}{r}\right) \tag{2.11}$$

Similarly,

$$r_2^2 \cong r^2\left(1 + \frac{2a\cos\theta}{r}\right) \tag{2.12}$$

Using the Binomial theorem and retaining terms upto the first order in a/r; we obtain,

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) \tag{2.13(a)}$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right) \tag{2.13(b)}$$

Using Eqs. (2.9) and (2.13) and $p = 2qa$, we get

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \tag{2.14}$$

Now, $p\cos\theta = \mathbf{p}.\hat{\mathbf{r}}$

where $\hat{\mathbf{r}}$ is the unit vector along the position vector **OP**.

The electric potential of a dipole is then given by

$$V = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p}.\hat{\mathbf{r}}}{r^2}; \qquad (r >> a) \tag{2.15}$$

Equation (2.15) is, as indicated, approximately true only for distances large compared to the size of the dipole, so that higher order terms in a/r are negligible. For a point dipole **p** at the origin, Eq. (2.15) is, however, exact.

From Eq. (2.15), potential on the dipole axis ($\theta = 0, \pi$) is given by

$$V = \pm \frac{1}{4\pi\varepsilon_0} \frac{p}{r^2} \tag{2.16}$$

(Positive sign for $\theta = 0$, negative sign for $\theta = \pi$.) The potential in the equatorial plane ($\theta = \pi/2$) is zero.

The important contrasting features of electric potential of a dipole from that due to a single charge are clear from Eqs. (2.8) and (2.15):

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector **r** and the dipole moment vector **p**. (It is, however, axially symmetric about **p**. That is, if you rotate the position vector **r** about **p**, keeping θ fixed, the points corresponding to P on the cone so generated will have the same potential as at P.)

(ii) The electric dipole potential falls off, at large distance, as $1/r^2$, not as $1/r$, characteristic of the potential due to a single charge. (You can refer to the Fig. 2.5 for graphs of $1/r^2$ versus r and $1/r$ versus r, drawn there in another context.)

2.5 Potential due to a System of Charges

Consider a system of charges $q_1, q_2,..., q_n$ with position vectors $\mathbf{r}_1, \mathbf{r}_2,...,$ $\mathbf{r}_n$ relative to some origin (Fig. 2.6). The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{1P}}$$

where r_{1P} is the distance between q_1 and P.

Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{2P}}, \quad V_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_3}{r_{3P}}$$

where r_{2P} and r_{3P} are the distances of P from charges q_2 and q_3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges

$$V = V_1 + V_2 + ... + V_n \tag{2.17}$$

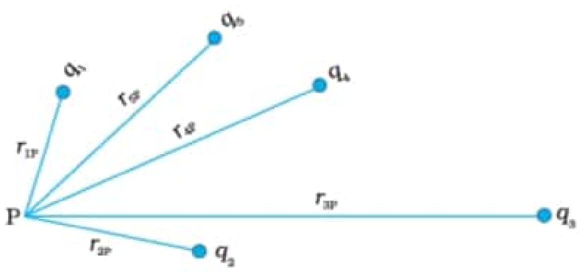

FIGURE 2.6 Potential at a point due to a system of charges is the sum of potentials due to individual charges.