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CLASS IX

MATHEMATICS

CHAPTER 1

NUMBER SYSTEMS

Real Numbers and their Decimal Expansions

1. Rational Numbers

If the rational number is in the form of a/b then by dividing a by b we can get two situations.

a. If the remainder becomes zero

While dividing if we get zero as the remainder after some steps then the decimal expansion of such number is called terminating.

b. If the remainder does not become zero

While dividing if the decimal expansion continues and not becomes zero then it is called non-terminating or repeating expansion.

2. Irrational Numbers

If we do the decimal expansion of an irrational number then it would be **non –terminating non-recurring** and vice-versa. i. e. the remainder does not become zero and also not repeated.

Example:

$\pi = 3.141592653589793238......$

Representing Real Numbers on the Number Line

To represent the real numbers on the number line we use the process of successive magnification in which we visualize the numbers through a magnifying glass on the number line.

Identities Related to Square Roots

1. $\sqrt{pq} = \sqrt{p}\sqrt{q}$
2. $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$
3. $(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$
4. $(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$
5. $(\sqrt{p} + \sqrt{q})(\sqrt{r} + \sqrt{s}) = \sqrt{pr} + \sqrt{ps} + \sqrt{qr} + \sqrt{qs}$
6. $(\sqrt{p} + \sqrt{q})^2 = p + 2\sqrt{pq} + q$

Laws of Exponents for Real Numbers

If we have a and b as the base and m and n as the exponents, then

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}, m > n$
4. $a^m b^m = (ab)^m$
5. $a^0 = 1$
6. $a^1 = a$
7. $1/a^n = a^{-n}$

- Let a > 0 be a real number and n a positive integer.

  Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$

Ex 1.1 Q 3) Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans. There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$

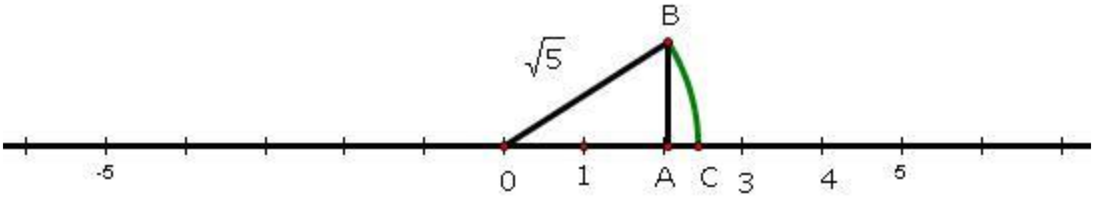
$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$. Therefore, rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

EX 1.2 Q 3) Show how $\sqrt{5}$ can be represented on the number line.

Ans- Using Pythagoras Theorem: $5=2^2+1^2$

Taking positive square root we get $\sqrt{5}=\sqrt{(2)^2+(1)^2}$



1. Mark a point 'A' representing 2 units on number line.
2. Now construct AB of unit length perpendicular to OA. Join OB
3. Now taking O as centre and OB as radius draw an arc, intersecting number line at point C.
4. Point C represents $\sqrt{5}$ on number line. [length (OB) = length (OC)]

EX 1.3 Q 3) Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Ans. (i) $0.\overline{6}=0.666...$

$x = 0.666\ldots$

$10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x=\frac{2}{3}$

(ii) $0.\overline{47}=0.4777.....$

$=\frac{4}{10}+\frac{0.777}{10}$

$x = 0.777\ldots$

$10x = 7.777\ldots$

$10x = 7 + x$

$x = \frac{7}{9}$

$$\frac{4}{10} + \frac{0.777\ldots}{10} = \frac{4}{10} + \frac{7}{90}$$
$$= \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001\ldots$

$x = 0.001001\ldots$

$1000x = 1.001001\ldots$

$1000x = 1 + x$

$999x = 1$

$x = \frac{1}{999}$

EX 1.3 Q 8) Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans.

$$\frac{5}{7} = 0.\overline{714285}$$
$$\frac{9}{11} = 0.\overline{81}$$

3 irrational numbers are as follows.

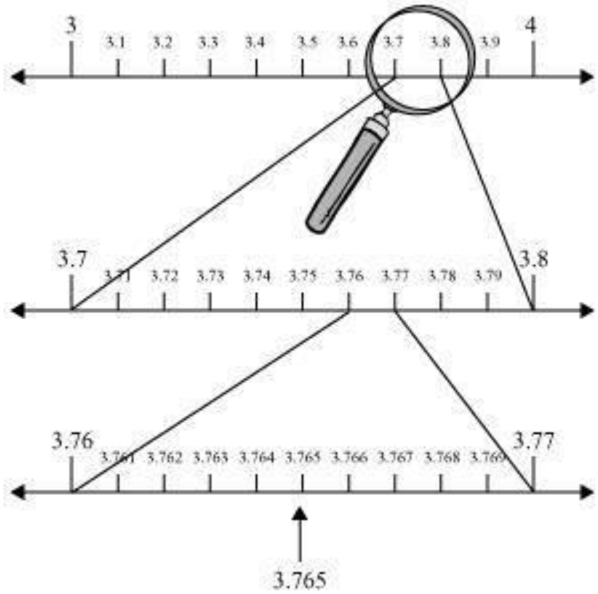
0.73073007300073000073…

0.75075007500075000075…

0.790790079000790000079…

EX 1.4 Q1) Visualise 3.765 on the number line using successive magnification.

3.765 can be visualised as in the following steps.



EX 1.5 Q 2 Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

Ans. (i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

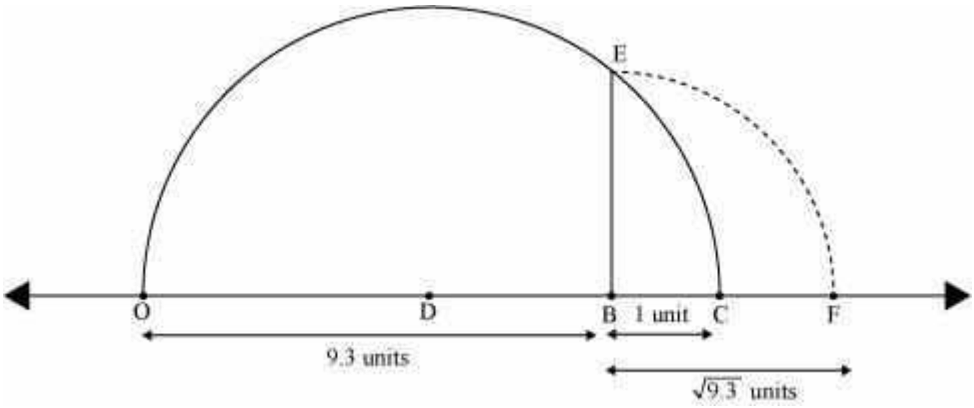
$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$ $=\left(3\right)^2-\left(\sqrt{3}\right)^2$

= 9 − 3 = 6

EX 1.5 Q4) Represent $\sqrt{9.3}$ on the number line.

Mark a line segment OB = 9.3 on number line. Take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



EX 1.5 Q5) Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Ans. (i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1 \quad (\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$$

EX 1.6 Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$

Ans. (i)

$$64^{\frac{1}{2}} = (2^6)^{\frac{1}{2}}$$

$$= 2^{6\times\frac{1}{2}} \qquad [(a^m)^n = a^{mn}]$$

$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$

$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^1 = 2$$

EX 1.6 Q 3) Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$

Ans. i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$

$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$